Solution to MHT CET – 2021
21st September (Shift - 1)

Section I

PHYSICS

1. (D)

Magnetic potential energy $U = \frac{1}{2}LI^2$

$\therefore L = \frac{2U}{I^2} = \frac{2\times25\times10^{-3}}{(50\times10^{-3})^2}$

$= 20\ H$

2. (B)

Frequency of a simple pendulum is given by

$n = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

$\therefore \frac{n'}{n} = \sqrt{\frac{g'}{g}}$

$g' = g\left(1-\frac{d}{R}\right) = g\left(1-\frac{1}{2}\right) = \frac{g}{2}$

$\therefore n' = \frac{n}{\sqrt{2}}$

3. (A)

$\gamma = \frac{C_p}{C_v}$, Also $C_p - C_v = R$

$C_p = \gamma C_v$

$\therefore \gamma C_v - C_v = R$

$\therefore C_v(\gamma - 1) = R$

$\therefore C_v = \frac{R}{\gamma - 1}$

4. (B)

$n_i = 1.6\times10^{16}\ m^{-3}$, $n_h = 4\times10^{22}\ m^{-3}$

$n_e n_h = n_i^2$

$\therefore n_e = \frac{n_i^2}{n_h} = \frac{(1.6\times10^{16})^2}{4\times10^{22}} = \frac{1.6\times1.6\times10^{32}}{4\times10^{22}}$

$= 6.4\times10^9\ m^{-3}$

5. (C)

Force due to electric field $\vec{F}_e = q\vec{E}$

Force due to magnetic field $\vec{F}_m = q\left(\vec{v}\times\vec{B}\right)$

$\therefore$ Total force $\vec{F} = q\vec{E} + q\left(\vec{v}\times\vec{B}\right)$

6. (C)

Initial frequency $\nu = 2\nu_0$

Final frequency $\nu' = \frac{2\nu_0}{3}$

ν' is less than the threshold frequency ν_0

Hence no photoelectrons will be emitted and photoelectric current will be zero.

7. (D)

$$\frac{1}{\alpha_{DC}} - \frac{1}{\beta_{DC}} = \frac{I_e}{I_c} - \frac{I_e}{I_c}$$

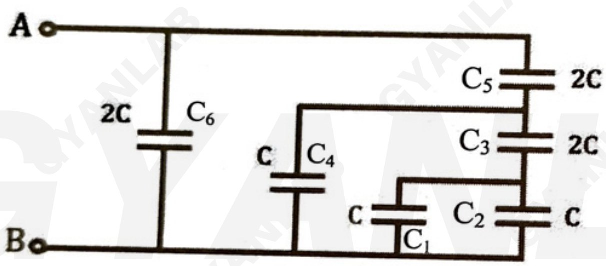
$$\frac{I_e - I_b}{I_c} = \frac{I_c}{I_c} = 1$$

8. (B)

Angular width of central maximum $\theta = \frac{2\lambda}{a}$

Where a is slit width

9. (C)



C_1 and C_2 are in parallel
Their equivalent capacitance $C_7 = 2C$
C_7 and C_3 are in series
Their equivlanet capacitance is $C_8 = C$
C_8 and C_4 are in parallel
Their equivalent capacitance is $C_9 = 2C$
C_9 and C_5 are in series. Their equivalent capacitance $C_{10} = C$
C_{10} and C_6 are in parallel. Their equivalent capacitance is 3C.
Hence equivalent capacitance between A and B is 3C.

10. (B)

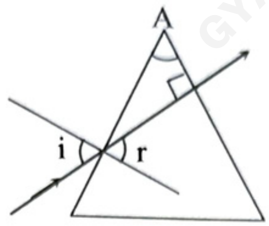
De–Broglie wavelength $\lambda \propto \frac{1}{p}$ where p is momentum

For an electron velocity $v \propto \frac{1}{n}$, hence $p \propto \frac{1}{n}$

$\therefore \frac{\lambda_1}{\lambda_4} = \frac{1}{4}$ $\therefore \lambda_1 = \frac{\lambda_4}{4}$

11. (D)
Magnetic field inside a solenoid is given by
$B = \mu_0 nI$
If n is doubled and I is made $\left(\frac{1}{3}\right)^{rd}$, the value of B will become $\frac{2}{3}$B.

12. (A)
From the figure A = r
$n = \frac{\sin r}{\sin r} = \frac{\sin i}{\sin A} = \frac{i}{A}$
(Angles are small)
$\therefore\ i = An$



13. (C)
$\frac{Q}{tA} = \frac{k\Delta\theta}{d}$
$\therefore\ k = \frac{Q}{tA} \cdot \frac{d}{\Delta\theta}$

$\frac{Q}{tA} = 900$ kcal per minute per $m^2 = \frac{900}{60} = 15$ kcal/sm^2
$d = 3\text{ cm} = 3\times10^{-2}\text{ m},\ \Delta\theta = 15°C$
$\therefore\ k = \frac{15\times3\times10^{-2}}{15} = 3\times10^{-2}$ kcal/ms°C

14. (D)
The diode is forward biased.
Potential difference = 5 − 3 = 2 V
$I = \frac{V}{R} = \frac{2}{200} = 10^{-2}\text{ A}$

15. (D)
Let v be the velocity of mass m and v′ be the velocity of mass M after collision.
By law of conservation of momentum
$mv = Mv'$
$\therefore\ \frac{v'}{v} = \frac{m}{M}$
Coefficient of restitutions = $\frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$
$= \frac{v'}{v}$

16. (B)
$C = \frac{KA\varepsilon_0}{d}$
$\therefore\ \frac{C_2}{C_1} = \frac{k_2}{k_1} \cdot \frac{d_1}{d_2}$

$\therefore \frac{6}{2} = \frac{k_2}{1} \cdot \frac{1}{2}$

$\therefore k_2 = 6$

17. (B)

By Wien's displacement law λT = constant

$\therefore \lambda \propto \frac{1}{T}$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_1} = \frac{2000}{3000} = \frac{2}{3}$

$\therefore \lambda_2 = \frac{2}{3}\lambda_1$

18. (A)

For isothermal process : $P_1V_1 = P_2V_2$

$\therefore P_2 = \frac{P_1V_1}{V_2} = \frac{P_1}{2} \qquad \left(\because \frac{v_1}{v_2} = \frac{1}{2}\right)$

For adiabatic process : $P_2V_2^{\gamma} = P_3V_3^{\gamma}$

$\therefore P_3 = P_2\left(\frac{V_2}{V_3}\right)^{\gamma} = P_2\left(\frac{2V_1}{V_3}\right)^{\gamma} = \frac{P_1}{2}\left(\frac{1}{8}\right)^{5/3} \qquad [\because \frac{V_1}{V_3} = \frac{1}{16}]$

$= \frac{P_1}{2} \cdot \frac{1}{32} = \frac{P_1}{64}$

19. (C)

$mr\omega^2 = \frac{k}{r}$

$\therefore \omega^2 = \frac{k}{mr^2} \qquad \therefore \omega^2 \propto \frac{1}{r^2}$

$\therefore \omega \propto \frac{1}{r}, \; T = \frac{2\pi}{\omega} \qquad \therefore T \propto r$

20. (A)

$E = e\sigma \cdot A(T^4 - T_0^4)$ and $A = \ell b$

When ℓ and b change to $\frac{\ell}{3}$ and $\frac{b}{3}$

$A' = \frac{A}{9}$

$\frac{E'}{E} = \frac{A'}{A}\frac{(327+273)^4}{(27+273)^4}; \qquad \frac{E'}{E} = \frac{1}{9}\left(\frac{600}{300}\right)^4$

$\therefore E' = \frac{1}{9} \times (2)^4 \times E \qquad \Rightarrow E' = \frac{16E}{9}$

21. (A)
For open organ pipe :
Fundamental frequency $n = \frac{V}{2L_o}$

First overtone $n_1 = 2n = \frac{V}{L_o}$

For closed organ pipe
Fundamental frequency $n' = \frac{V'}{4L_c}$

First overtone $n_1' = 3n' = \frac{3V'}{4L_c}$

$n_1 = n_1' \quad \therefore \frac{V}{L_o} = \frac{3V'}{4L_c}$

$\therefore \frac{L_o}{L_c} = \frac{4}{3}\frac{V}{V'}$ ...(1)

$V = \sqrt{\frac{k}{\rho_2}}, \; V' = \sqrt{\frac{k}{\rho_1}}$

where k is the adiabatic bulk modulus, which is reciprocal of compressibility.

$\therefore \frac{V}{V'} = \sqrt{\frac{\rho_1}{\rho_2}}$

Putting this value in Eq.(1)

$\frac{L_o}{L_c} = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

22. (A)

23. (C)
$n = 50$ Hz, $t = 0.01$ s
If T is the period and phase difference is ϕ
then , $\frac{t}{T} = \frac{\phi}{2\pi}$

$\therefore \phi = 2\pi\frac{t}{T} = 2\pi nt$

$\therefore \phi = 2\pi \times 50 \times 0.01 = \pi$ rad

24. (B)
Temperature in kelvin $= -197 + 273 = 76$ K

25. (C)

26. (D)
In isobaric process pressure remains constant.

27. (B)
Maximum particle velocity $= Y_0\omega = 2\pi n\, Y_0$

Wave velocity = $n\lambda$

$\therefore \quad \frac{2\pi n Y_0}{8} = n\lambda$

$\therefore \quad \lambda = \frac{\pi Y_0}{4}$

28. (B)

$\frac{5}{R} = \frac{\ell_1}{100\ell_1}$ ...(1)

$\frac{5\times 2}{R} = \frac{1.6\ell_1}{100-1.6\ell}$

$\therefore \quad \frac{5}{R} = \frac{0.8\ell_1}{100-1.6\ell_1}$ ...(2)

Equating (1) and (2) and solving we get $\ell_1 = 25\,\text{cm}$

Putting this in Eq.(1), we get $R = 15\ \Omega$

29. (C)

Since the current and emf are in phase the circuit is purely resistive.

30. (B)

$I = 1.2\ \text{kg–m}^2$, K.E. = 1500 J, $\alpha = 25\ \text{rad/s}^2$

$K.E. = \frac{1}{2}I\omega^2$

$\therefore \quad 1500 = \frac{1}{2}\times 1.2\times\omega^2$

$\therefore \quad \omega^2 = \frac{2\times 1500}{1.2} = 2500 \qquad \therefore \quad \omega = 50\ \text{rad/s}$

$\omega = \omega_0 + \alpha t$

$\therefore \quad 50 = 0 + 25t = 25t \qquad \therefore \quad t = 2\ s$

31. (B)

$\sin 2\theta = 2\sin\theta\,.\,\cos\theta$

$\therefore \quad \sin\theta\cos\theta = \frac{\sin 2\theta}{2}$

$V = 80\sin(100\pi t)\cos(100\pi t)$

$= \frac{80\sin 200\pi t}{2} = 40\sin 200\pi t$

Amplitude or the peak value of the voltage = 40 V

32. (D)

$g' = g\frac{R^2}{(R+h)^2} \qquad g' = \frac{g}{3}$

$\therefore \quad \frac{1}{3} = \frac{R^2}{(R+h)^2} \qquad \therefore \quad \frac{1}{\sqrt{3}} = \frac{R}{R+h}$

$\therefore \quad \sqrt{3}R = R + h$

$\therefore \quad h = \left(\sqrt{3}-1\right)R$

33. (A)

$$21\times\frac{\lambda_1 D}{d}=N\times\frac{\lambda_2 D}{d}$$

$$\therefore\ N=21\frac{\lambda_1}{\lambda_2}=\frac{21\times4800}{5600}=18$$

34. (A)

If the wire falls through a height h, the velocity acquired by it is

$$v=\sqrt{2g\ell}$$

The emf induced in the wire

$$e=BLv=BL\sqrt{2g\ell}$$

$$\therefore\ \text{Current } I=\frac{e}{R}=\frac{BL\sqrt{2g\ell}}{R}$$

35. (A)

$$\frac{R_A}{R_B}=\frac{40}{60}=\frac{2}{3}$$

$$R=\rho\cdot\frac{\ell}{A}=\rho\cdot\frac{\ell}{\pi r^2}$$

Length of the wires are same.

$$\therefore\ \frac{R_A}{R_B}=\frac{\rho_A}{\rho_B}\left(\frac{r_B}{r_A}\right)^2$$

$$\therefore\ \frac{2}{3}=\frac{\rho_A}{\rho_B}\left(\frac{1}{3}\right)^2=\frac{\rho_A}{\rho_B}\cdot\frac{1}{9}$$

$$\therefore\ \frac{\rho_A}{\rho_B}=6$$

36. (A)

Let x be the real depth when the apparent depth is 10 cm.

$$\therefore\ \mu=\frac{x}{10}$$

Let y be the real depth when the apparent depth is 6 cm

$$\therefore\ \mu=\frac{y}{6}$$

$$\therefore\ \frac{x}{10}=\frac{y}{6}\qquad\therefore\ x=\frac{5}{3}y$$

Also $x+y=24$ $\quad\therefore\ \frac{5}{3}y+y=24$ $\quad\therefore\ \frac{8}{3}y=24$

$\therefore\ y=9$ cm

$$\mu=\frac{y}{6}=\frac{9}{6}=1.5$$

37. (C)

Relative permeability $\mu_r=1+\chi$

$$\therefore\ \chi=\mu_r-1=0.85-1=-0.15$$

38. (D)

In fundamental mode the length of the closed tube is $\frac{\lambda}{4}$

Time required to travel a distance $\frac{\lambda}{4}$ is t.

Hence time required to travel a distance λ will be 4t.

$\therefore$ Time period $T = 4t$;

Frequency $n = \frac{1}{T} = \frac{1}{4t} = \frac{0.25}{t}$

39. (A)

For the particle to escape from earth's gravitational field, it should be given kinetic energy equal to its binding energy.

$$\therefore \quad \frac{1}{2}mv^2 = \frac{GMm}{4R}$$

$$\therefore \quad V = \left[\frac{GM}{2R}\right]^{1/2}$$

40. (B)

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

$$\therefore \quad \frac{F_2}{F_1} = \left(\frac{r_1}{r_2}\right)^2 = (2)^2 = 4$$

$$\therefore \quad F_2 = 4F_1$$

41. (D)

Magnetic field at the centre $B = \frac{\mu_o I}{R}$

ϕ = Magnetic flux passing through the smaller coil = $\pi r^2 B$

$$\therefore \phi = \pi r^2 \times \frac{\mu_o I}{R}$$

$$\therefore \quad M = \frac{\phi}{I} = \frac{\mu \pi r^2}{R} \qquad \therefore \quad M \propto \frac{r^2}{R}$$

42. (A)

Series limit of Lyman series is given by

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1} - \frac{1}{\infty}\right) = R$$

Series limit of Balmer series is given by

$$\frac{1}{\lambda_3} = R\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{R}{4}$$

First line of Lyman series is given by

$$\frac{1}{\lambda_2} = R\left(\frac{1}{1} - \frac{1}{4}\right) = R - \frac{R}{4} = \frac{1}{\lambda_1} - \frac{1}{\lambda_3}$$

$$\therefore \quad \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$$

43. (D)
Since the velocity is constant, the net force acting on the ball is zero. Forces acting are
Weight of ball = Mg (downwards)
Upthrust = weight of water displaced
= (volume of water) × (density of water) × g
= (volume of ball) × (density of water) × g
$= \frac{M}{d_1} \times d_2 \times g = Mg\frac{d_2}{d_1}$ (upwards)

F = viscous force (upwards)

$\therefore F + Mg\frac{d_2}{d_1} = Mg$

$\therefore F = Mg\left(1 - \frac{d_2}{d_1}\right)$

44. (D)
Gravitational potential energy gained by the ball = Elastic potential energy in the spring

$mgh = \frac{1}{2}Fx$

$\therefore h = \frac{Fx}{2mg}$

$= \frac{5 \times 0.2}{2 \times 25 \times 10^{-3} \times 10}$

$= 2m$

45. (C)
The plane is flying horizontally. Hence initial vertical component of the velocity is zero.
If it reaches the ground in time t, then

$h = \frac{1}{2}gt^2 \qquad \therefore t^2 = \frac{2h}{g} = \frac{2 \times 980}{9.8} = 200$

$\therefore t = 10\sqrt{2}$ s

The horizontal component of the velocity is

$V = 200$ km/hr $= 200 \times \frac{5}{18} = \frac{1000}{18}$ m/s

The horizontal distance to be covered is

$d = Vt = \frac{1000}{18} \times 10\sqrt{2} = \frac{10^4}{9\sqrt{2}}$ m

46. (A)

Initial current $I = \frac{V}{z}$, Final current $I' = \frac{V}{z}$

$\frac{1}{2} = \frac{I'}{I} = \frac{z}{z'} \qquad \therefore \frac{z}{z'} = \frac{1}{2}$

$\therefore \frac{z^2}{z'^2} = \frac{1}{4} \qquad \therefore \frac{R^2 + X_c^2}{R^2 + X_c'^2} = \frac{1}{4}$

$\therefore 4R^2 + 4X_c^2 = R^2 + X_c'^2$

$\therefore 3R^2 = X_c'^2 - 4X_c^2$ ...(1)

$X_c = \frac{1}{\omega C}$, $X' = \frac{3}{\omega C}$ $\therefore X' = 3X_c$

Putting this value of X' in (1) we get

$3R^2 = 9X_c^2 - 4X_c^2 = 5X_c^2$

$\therefore \frac{X_c^2}{R^2} = \frac{3}{5} = 0.6$

$\therefore \frac{X_c}{R} = \sqrt{0.6}$

47. (B)

$d = 1\text{ mm} = 10^{-3}\text{ m}$, $D = 1.33\text{ m}$

$\lambda = 6300\ \text{Å} = 6.3 \times 10^{-7}\text{ m}$

$\lambda_w =$ wavelength in water $= \frac{6.3\times10^{-7}}{1.33}\text{ m}$

Fringe width $X = \frac{\lambda_w D}{d} = \frac{6.3\times10^{-7}\times1.33}{1.33\times10^{-3}} = 6.3\times10^{-4}\text{ m}$

48. (C)

Total surface energy before coalesce

$E_1 = 2(4\pi R^2)T$

But $\frac{4}{3}\pi R^2 \times 2 = \frac{4}{3}\pi R'^3$

Total surface energy after coalesce

$E_2 = 4\pi R'^2 T = 4\pi 2^{2/3} R^2 T$

$\therefore \frac{E_1}{E_2} = \frac{2(4\pi R^2)T}{4\pi 2^{2/3} R^2 T} = 2^{1-\frac{2}{3}} = 2^{1/3}$

49. (D)

The kinetic energy of the photo electrons and hence the stopping potential does not depend on the intensity of light.

50. (C)

$f = 8$ cm, when the particle is at mean position, $u = -14$ cm

$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{8} - \frac{1}{14} = \frac{3}{56}$

$\therefore v = \frac{56}{3} \approx 19$ cm

When the particle is at one of the extreme positions its distance from the lens is $14 + 1 = 15$ cm

$\therefore u = -15$ cm

Again, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{8} - \frac{1}{15} = \frac{7}{120}$

$\therefore v = \frac{120}{7} \approx 17$ cm

Amplitude of the image $= 19 - 17 = 2$ cm

CHEMISTRY

51. (B)
$K = {}^\circ C + 273.15 \quad = -197\ {}^\circ C + 273.15 = 76.15\ K$

52. (A)
Starch is a polymer of α-D- glucose.

53. (D)

54. (B)

$$\underset{\displaystyle \underset{|}{\ } \atop CH_3}{H_2C = CH - \overset{2^\circ}{C}H - OH}$$

Secondary allylic alcohol

55. (C)
$3x \rightarrow 2y + z$

Rate of formation $= -\frac{1}{3}\frac{d[x]}{dt} = \frac{1}{2}\frac{d[y]}{dt} = \frac{d[z]}{dt}$

$\frac{d[z]}{dt} = \frac{1}{3}(0.072) = 0.024\ mol\ s^{-1}$

56. (A)
A plot of rate versus $[A]_t$ is a straight line passing through origin. This is shown in Fig. The slope of straight line = k.

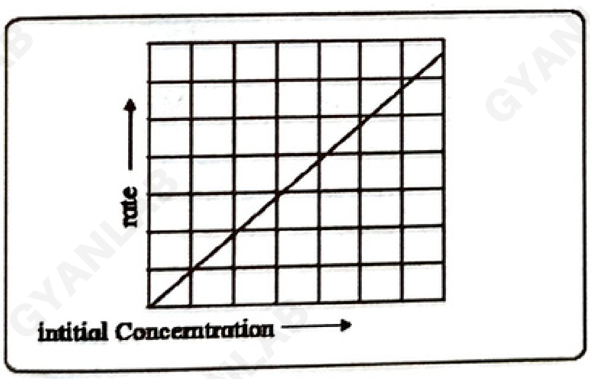
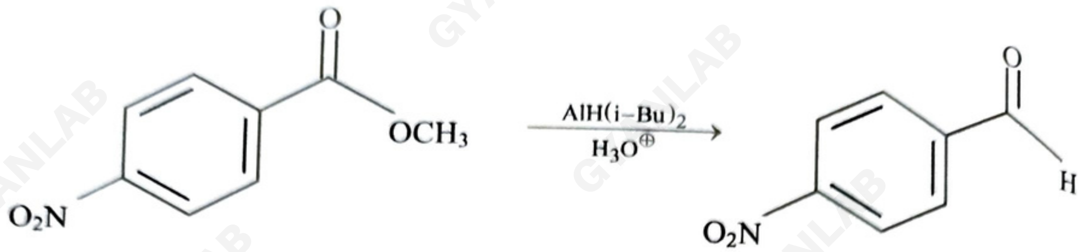


Fig. : Variation of rate with [A].

57. (B)

$$\underset{\text{Ethanoyl Chloride}}{CH_3-\overset{O}{\overset{||}{C}}-Cl} \xrightarrow{H_2O} \underset{\text{Ethanoic acid}}{CH_3-\overset{O}{\overset{||}{C}}-OH} + HCl$$

58. (C)



59. (C)

Physisorption is reversible. Option A, B and D are characteristics of chemisorption.

60. (C)

Pressure is an intensive property while others are extensive properties.

61. (A)

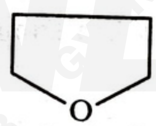
Conductivity is inversely proportional to resistivity i.e. $k = \frac{1}{\rho}$

62. (B)

NH_4CN – Basic, NH_4F – Slightly acidic,
CH_3COONa – Basic, CH_3COONH_4 – Neutral
The solution of NH_4F is only slightly acidic and turns blue litmus red.

63. (D)

It is heterocyclic nonaromatic compound.



Tetrahydrofuran

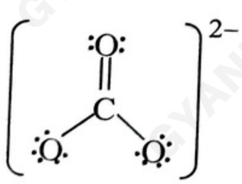
64. (A)

Ionic character of metal halides decreases in the order : MF > MCl > MBr > MI

65. (C)

$\mu = \sqrt{n(n+2)}$ BM
For n = 1, $\mu = \sqrt{3} = 1.73$ BM

66. (D)



Formal charge on C atom $= \text{V.E.} - \text{N.E.} - \frac{1}{2}\text{B.E.}$

$= 4 - 0 - \frac{1}{2}(8)$

$= 0$

67. (D)

$$\underset{0.02\,M}{NaOH} \longrightarrow Na^+ + \underset{0.02\,M}{OH^-}$$

$$p^{OH} = -\log_{10}[OH^-] = -\log_{10}[0.02]$$
$$= -\log_{10}(2 \times 10^{-2}) = 2 - 0.3010$$
$$p^{OH} = 1.6990$$
$$\therefore\ p^H = 14 - P^H = 14 - 1.6990 = 12.301$$

68. (A)
The boiling point of methyl halides decreases in the order :
$CH_3I > CH_3Br > CH_3Cl > CH_3F$
(With decrease in atomic mass of halogen)

69. (D)
The change in internal energy is given by, $\Delta U = Q + W$
As container is insulated, $Q = 0$

$$\therefore\ \Delta U = W = -P_{ext}\,\Delta V = -P_{ex}\,(V_2 - V_1)$$
$$\therefore\ \Delta U = -2.5\ \text{atm}\ (4.5\ L - 2.5\ L)$$
$$= -5.0\ \text{L.atm}$$
$$= -5.0 \times 101.325\ J$$
$$\therefore\ \Delta U = -506.625\ J$$

70. (C)
Ethanol + Acetone show positive deviation from Raoult's law while other solutions show negative deviations.

71. (C)
$NH_4^{\oplus}$ is best stabilized by solution while the stabilization by solvation is very poor in $R_3NH^{\oplus}$.
Order of stabilization: $NH_4^{\oplus} > R-NH_3^{\oplus} > R_2NH^{\oplus} > R_3NH^{\oplus}$

72. (B)

$$n_2 = 0.1\ \text{mol},\ n_1 = \frac{16.2}{18} = 0.9\ \text{mol}$$
$$x_1 = \frac{n_1}{n_1 + n_2} = \frac{0.9}{0.1 + 0.9} = 0.9$$
$$P_1 = P_1^0 x_1 = 32\ \text{mm Hg} \times 0.9$$
$$= 28.8\ \text{mm Hg}$$

73. (D)
The percentage efficiency of packing in BCC structure is 68%.

74. (D)
SO_3 – Acidic, Na_2O – Basic
N_2O – Neutral, Al_2O_3 – Amphoteric

75. (A)
Teflon is homopolymer while others are copolymers.

76. (C)

$c = 0.4$ M, $\wedge = 2.5 \times 10^5\ \Omega^{-1}\ cm^2\ mol^{-1}$, $\rho = ?$

(i) $\wedge = \dfrac{1000k}{C}$ $\quad \therefore\ k = \dfrac{\wedge C}{1000}$ $\quad \therefore\ k = \dfrac{2.5 \times 10^5 \times 0.4}{1000} = 100$

(ii) $k = \dfrac{1}{\rho}$ $\quad \therefore\ \rho = \dfrac{1}{k} = \dfrac{1}{100} = 1 \times 10^{-2}$

77. (C)

78. (C)

Clemmensen reduction

$$>C=O + 4[H] \xrightarrow[\Delta]{Zn-Hg,\ Conc.HCl} >CH_2 + H_2O$$

(Carbonyl group in aldehydes and ketones) (Methylene group)

79. (B)

$$\underset{\text{Alkyl chloride}}{R-Cl} + NaI \xrightarrow{\text{acetone}} \underset{\text{Alkyl iodide}}{R-I} + NaCl\downarrow$$

This reaction is known as Finkelstein reaction.

80. (C)

Secondary amines are the strongest bases.

$\therefore$ $(CH_3)_2NH$ acts as strong base.

81. (A)

Thiocyanate (SCN^-) is a neutral ligand.

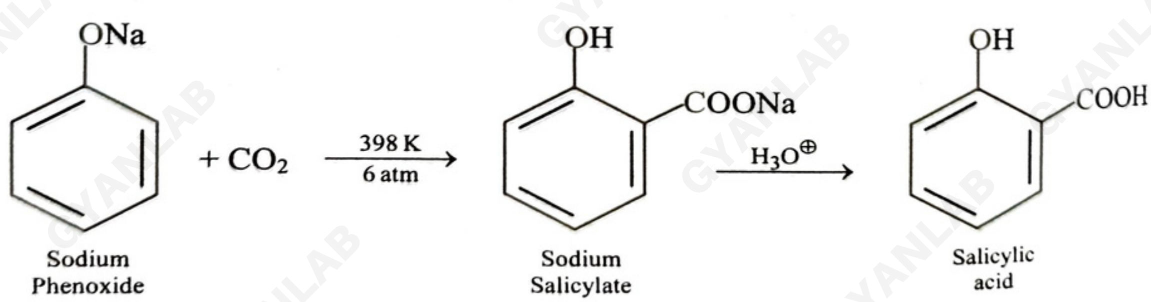
82. (B)

$\underset{\text{Ethane}}{C_2H_6} \rightarrow$ Molar mass = 30 g mol^{-1}

30 g of ethane = 22.4 dm^3 at STP

$\therefore$ 75 g of ethane $= \dfrac{22.4 \times 75}{30} = 56.0\ dm^3$

83. (A)



84. (B)

$\lambda = 460$ nm $= 460 \times 10^{-9}$ m, $\quad \nu = ?$

$\nu = \dfrac{c}{\lambda} = \dfrac{3 \times 10^8\ m\,s^{-1}}{460 \times 10^{-9}\ m} = 6.5 \times 10^{14}$ Hz

85. (C)

$$\underset{\text{Ethene}}{CH_2=CH_2} + H_2 \xrightarrow[\Delta]{\text{Ni(finely divided)}} \underset{\text{Ethane}}{CH_3-CH_3}$$

86. (B)

Dodecane – $C_{12}H_{26} \Rightarrow CH_3-(CH_2)_{10}-CH_3$

87. (A)

Compounds of unipositive ions of alkali metals are diamagnetic.

88. (B)

$a = 3.86$ Å $= 3.86 \times 10^{-8}$ cm, $r = ?$

For simple cubic close structure,

$$r = \frac{a}{2} = \frac{3.86\times10^{-8}}{2} = 1.93 \times 10^{-8}\ \text{cm}$$

89. (B)

$[A]_0 = 1$, $[A]_t = \frac{1}{8}$, $t = 23.03$ min

For first order reaction,

(i) $k = \frac{2.303}{t}\log_{10}\frac{[A]_0}{[A]_t}$

$= \frac{2.303}{23.03\ \text{min}}\log_{10}\frac{1}{(1/8)}$

$= \frac{2.303}{23.03}\times\log_{10}8 = \frac{2.303\times0.9031}{23.03}$

$= 0.0903\ \text{min}^{-1}$

(ii) $t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.0903\ \text{min}^{-1}} = 7.7$ min

90. (D)

$m = 0.1$ m, $K_b = 0.52°C\ \text{kg mol}^{-1}$

$\Delta T_b = K_b \times m = 0.52°C\ \text{kg mol}^{-1} \times 0.1\ \text{mol kg}^{-1} = 0.052\ °C$

$\Delta T_b = T_b - T_b^0 \quad \therefore\ T_b = \Delta T_b + T_b^0$

$\therefore\ T_b = 0.052 + 100 = 100.052\ °C$

91. (D)

$V_1 = 2.5 \times 10^{-2}\ m^3 = 25\ dm^3$, $V_2 = 1.3 \times 10^{-2}\ m^3 = 13\ dm^3$

$P_{ext} = 4.05$ bar, $W = ?$

$W = -P_{ext}\,\Delta V = -P_{ext}(V_2 - V_1)$

$= -4.05\ \text{bar} \times (13 - 25)\ dm^3$

$= 48.6\ dm^3\ \text{bar}$

$= 48.6 \times 100$ J

$\therefore\ W = 4860$ J

92. (A)

$[Co(NH_3)_6]Cl_3$ → Here, only one type of ligand i.e. NH_3 surrounds the Co^{3+} ion. Hence, it is homoleptic complex.

93. (B)

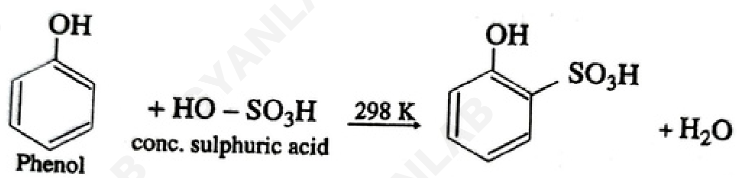
$K_a = 1.32 \times 10^{-5}$, $c = 0.05$ M, $\propto = ?$

$$\propto = \sqrt{\frac{Ka}{c}} = \sqrt{\frac{1.32\times10^{-5}}{0.05}} = \sqrt{2.64}\times10^{-2}$$

$$\therefore \propto = 1.62 \times 10^{-2}$$

94. (B)

When phenol is treated with conc. H_2SO_4 at room temperature (about 298 K), o-phenol sulphonic acid is formed.



o-phenol sulphonic acid
(2-Hydroxybenzene sulphonic acid)

95. (A)

$$CH_3-CH_2-CH_3 \xrightarrow[\text{uv light}]{Br_2} \underset{\substack{\text{1-Bromopropane}\\(3\%)}}{CH_3-CH_2-CH_2-Br} + \underset{\substack{\text{2-Bromopropane}\\(97\%)}}{CH_3-\underset{\substack{|\\Br}}{CH}-CH_3}$$

The ease of replacement of hydrogen atoms from the carbon is in the order of $3° > 2° > 1°$.

96. (B)

Potassium dichromate → $K_2Cr_2O_7$

Consider, oxidation state of Cr is x.

$K_2Cr_2O_7 \rightarrow 2(+1) + 2(x) + 7(-2) = 0$

$\therefore\ 2 + 2x - 14 = 0$

$2x = 12$

$\therefore\ x = +6$

$\therefore$ Oxidation state of Cr = +6

97. (C)

$M = 25\ g\ mol^{-1}$, $\rho = 3\ g\ cm^{-3}$,

For BCC structure, n = 2

Volume of unit cell $(a^3) = ?$

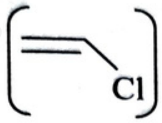
$$M = \frac{\rho \cdot a^3 N_A}{n}$$

$$\therefore\ a^3 = \frac{M \times n}{\rho \times N_A}$$

$$\therefore\ a^3 = \frac{25\ g\ mol^{-1} \times 2\ atoms}{3\ g\ cm^{-3} \times 6.022 \times 10^{23}\ atoms\ mol^{-1}}$$

$$= \frac{50 \times 10^{-23}}{18.066} = 2.76 \times 10^{-23}\ cm^3$$

98. (D)

Monomer → Vinyl chloride 

PVC (Polyvinyl chloride) structure → $\left[CH_2 - \underset{}{\overset{Cl}{\overset{|}{C}}}H \right]_n$

99. (C)

$I = 1A,\ t = 16.1\ min = 16.1 \times 60\ s$

$$\text{Moles of electrons actually passed} = \frac{Q(C)}{96500(C/mol\ e^-)}$$

$$= \frac{I \times t}{96500}$$

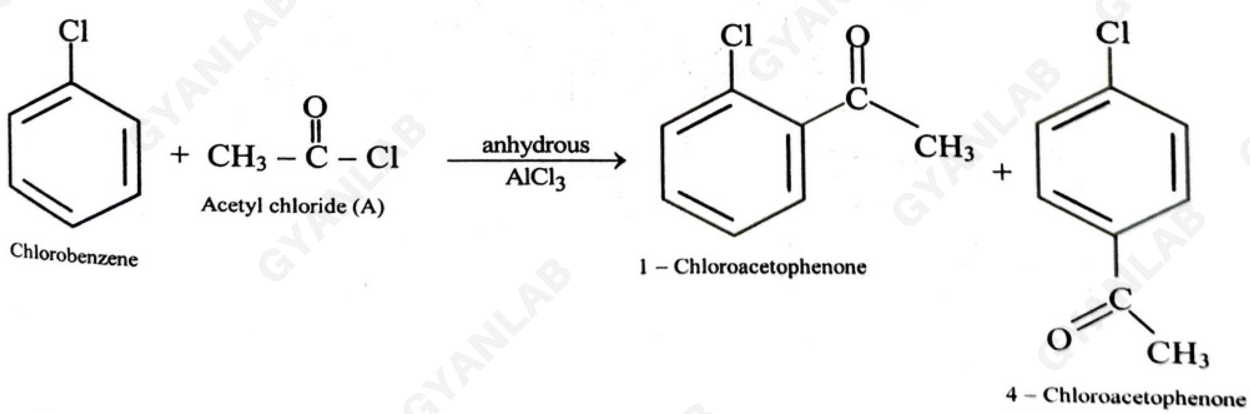
$$= \frac{1 \times 16.1 \times 60}{96500}$$

$$= 0.01\ mol$$

Now, $1\ mol = 6.022 \times 10^{23}$ electrons

$\therefore\ 0.01\ mol = 6.022 \times 10^{21}$ electrons

100. (D)



Section II

MATHEMATICS

101.(B)

$$2\sin\left(\theta+\frac{\pi}{3}\right) = \cos\left(\theta-\frac{\pi}{6}\right)$$

$$2\left[\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\right] = \left(\cos\theta\cos\frac{\pi}{6}+\sin\theta\sin\frac{\pi}{6}\right)$$

$$\therefore\ 2\left(\frac{\sin\theta}{2}+\frac{\sqrt{3}\cos\theta}{2}\right) = \left(\frac{\sqrt{3}\cos\theta}{2}+\frac{\sin\theta}{2}\right)$$

$$\therefore\ \left(\frac{\sin\theta}{2}+\frac{\sqrt{3}\cos\theta}{2}\right) = 0 \Rightarrow \sin\theta+\sqrt{3}\cos\theta = 0 \Rightarrow \tan\theta = -\sqrt{3}$$

102.(C)

$$\text{Let } I = \int[1+2\tan x(\tan x+\sec x)]^{1/2}dx = \int\left(1+2\tan^2x+2\tan x\sec x\right)^{1/2}dx$$

$$= \int\left[\left(1+\tan^2x\right)+\tan^2x+2\sec x\tan x\right]^{1/2}dx = \int\left(\sec^2x+\tan^2x+2\sec x\tan x\right)^{1/2}dx$$

$$= \int\left[(\sec x+\tan x)^2\right]^{1/2}dx = \int(\sec x+\tan x)\,dx = \int\sec x\,dx+\int\tan x\,dx$$

$$= \log|\sec x+\tan x) - \log|\cos x| + c = \log\frac{|\sec x+\tan x|}{|\cos x|}+c$$

$$= \log|\sec x(\sec x+\tan x)| + c$$

103.(D)

$$(3xy+y^2)\,dx+(x^2+xy)\,dy = 0$$

$$\therefore\ \frac{dy}{dx} = -\frac{\left(3xy+y^2\right)}{\left(x^2+xy\right)}$$

$$\text{Put } y = vx \Rightarrow \frac{dy}{dx} = v+x\frac{dv}{dx}$$

$$\therefore\ v+x\frac{dv}{dx} = -\frac{\left(3vx^2+v^2x^2\right)}{\left(x^2+vx^2\right)} = -\frac{3v+v^2}{1+v}$$

$$\therefore\ x\frac{dv}{dx} = -\frac{3v+v^2}{1+v}-v = -\frac{\left(2v^2+4v\right)}{1+v}$$

$$\therefore\ \int\frac{1+v}{\left(2v^2+4v\right)}dv = \int\frac{-1}{x}dx$$

$$\therefore\ \frac{1}{2}\int\frac{v+1}{v^2+2v}dv = -\int\frac{dx}{x} \Rightarrow \frac{1}{4}\int\frac{2(v+1)}{v^2+2v}dv = -\log x+\log c_1$$

$$\therefore\ \frac{1}{4}\log\left(v^2+2v\right)+\log x = \log c_1 \Rightarrow \frac{1}{4}\log\left(\frac{y^2}{x^2}+\frac{2y}{x}\right)+\log x = \log c_1$$

$\therefore \frac{1}{4}\log\left(\frac{y^2+2xy}{x^2}\right)+\log x = \log c_1 \Rightarrow \log\left(\frac{y^2+2xy}{x^2}\right)+\log x^4 = 4\log c_1$

$\therefore \log\left[\left(\frac{y^2+2xy}{x^2}\right)(x^4)\right] = \log c_1^4 \Rightarrow \log\left[x^2\left(y^2+2xy\right)\right] = \log c_1^4$

$\therefore x^2(y^2+2xy) = c$

104.(B)

From the given data, we write

x_i	p_i	x_ip_i	$p_ix_i^2$
0	0.4	0	0
1	0.6	0.6	0.6
Total		0.6	0.6

Variance $= \Sigma p_ix_i^2 - (\Sigma p_ix_i)^2 = 0.6-(0.6)^2 = 0.6-0.36 = 0.24$

105.(D)

The distance between given parallel lines

$$= \left|\frac{\left[(\hat{i}-2\hat{i})+(-\hat{j}+\hat{j})+(2\hat{k}-\hat{k})\right]\times(2\hat{i}+\hat{j}-2\hat{k})}{|2\hat{i}+\hat{j}-2\hat{k}|}\right| = \left|\frac{(-\hat{i}+\hat{k})\times(2\hat{i}+\hat{j}-2\hat{k})}{\sqrt{(2)^2+(1)^2+(-2)^2}}\right|$$

$$(-\hat{i}+\hat{k})\times(2\hat{i}+\hat{j}-2\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 0 & 1\\ 2 & 1 & -2\end{vmatrix} = \hat{i}(0-1)-\hat{j}(2-2)+\hat{k}(-1-0) = -\hat{i}-\hat{k}$$

$$\therefore d = \left|\frac{|-\hat{i}-\hat{k}|}{\sqrt{9}}\right| = \frac{\sqrt{2}}{3} \text{ units}$$

106.(D)

$e^{-y}.y = x$

$\therefore \frac{y}{e^y} = x \Rightarrow y = x\,e^y$ ...(1) and $e^y = \frac{y}{x}$ ...(2)

Now $y = xe^y$

$\therefore \frac{dy}{dx} = xe^y\frac{dy}{dx}+e^y$

$\therefore \frac{dy}{dx}(xe^y-1) = -e^y \Rightarrow \frac{dy}{dx} = \frac{-e^y}{xe^y-1}$

From (1) and (2), we write

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)\times\frac{1}{y-1} = \frac{-y}{x(y-1)} = \frac{y}{x(1-y)}$$

107.(D)

Lines given by $ax^2 + 2hxy + by^2 = 0$ make inclinations α and β.

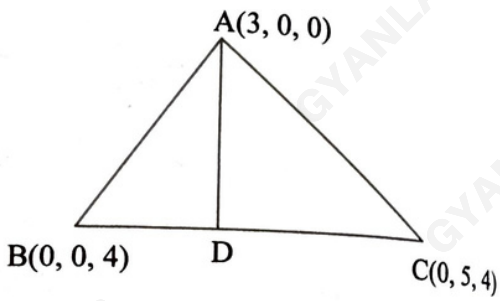
$\therefore \tan\alpha + \tan\beta = \frac{-2h}{b}$ and $\tan\alpha\tan\beta = \frac{a}{b}$

Now $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\left(-\frac{2h}{b}\right)}{1-\left(\frac{a}{b}\right)} = \frac{-2h}{b} \times \frac{b}{(b-a)}$

$\therefore \tan(\alpha+\beta) = \frac{-2h}{b-a} = \frac{2h}{a-b}$

108.(B)

Refer figure



Let AD be the angle bisector of angle A which divides BC in the ratio AB : AC.

Here $AB = \sqrt{9+16} = \sqrt{25}$ and

$AC = \sqrt{9+25+16}$

$= \sqrt{50}$

$\therefore$ D divides BC in the ratio $\sqrt{25} : \sqrt{50}$ i.e. 1 : 2

$\therefore$ Position vector of D $= \frac{(4)(2)\hat{k} + 5\hat{j} + 4\hat{k}}{1+2} = \frac{5j + 12\hat{k}}{3}$

109.(B)

We have lines

$\frac{x-1}{-3} = \frac{7(y-2)}{2\lambda} = \frac{z-3}{2}$ and $\frac{7(1-x)}{3\lambda} = \frac{y-5}{1} = \frac{z-6}{-5}$

i.e. $\frac{x-1}{-3} = \frac{y-2}{\left(\frac{2\lambda}{7}\right)} = \frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$

Since given lines are at right angles, we write

$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + (2)(-5) = 0$

$\therefore \frac{9\lambda}{7} + \frac{2\lambda}{7} - 10 = 0 \Rightarrow 11\lambda = 70 \Rightarrow \lambda = \frac{70}{11}$

110.(B)

Probability of finding defective bulb $= \frac{10}{100} = 0.1$

Let $p = 0.1 \Rightarrow q = 0.9$

We have $n = 5$ and to get required probability $x = 0$ and $x = 1$

$\therefore p = {}^5C_0(0.1)^0(0.9)^5 + {}^5C_1(0.1)^1(0.9)^4$

$= (0.9)^5 + (5)(0.1)(0.9)^4$

$= (0.9)^4[0.9 + 0.5] = (0.9)^4(1.4)$

$= (0.81)(0.81)(1.4) = (0.6561)(1.4) = 0.91854$

Note : Students can guess the correct option by approximation as follows.

(0.81) (0.81) (1.4) ≅ (0.64) (1.4) ≅ 0.896

111.(B)

Let (h, 0) be the centre of the circle and 'r' be the radius.

$\therefore (x-h)^2+(y-0)^2=r^2 \Rightarrow \quad (x-h)^2+y^2=r^2$ ...(1)

Differentiating w.r.t. x, we get

$\therefore 2(x-h)(1)+2y\frac{dy}{dx}=0 \Rightarrow h=x+y\frac{dy}{dx}$

Substituting value of 'h' in eq. (i), we get

$$y^2\left(\frac{dy}{dx}\right)^2+y^2=r^2 \Rightarrow y^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=r^2$$

Differentiating w.r.t. x, we get

$$y^2\left(2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\right)+\left[1+\left(\frac{dy}{dx}\right)^2\right](2y)\frac{dy}{dx}=0$$

$$\therefore y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$$

112.(D)

$$\frac{\cos(A+B)}{\cos(A-B)}=\frac{\sin(C+D)}{\sin(C-D)}$$

By Componendo – Dividendo, we wirte

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{\sin(C+D)+\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\frac{2\cos A\cos B}{-2\sin A\sin B}=\frac{2\sin C\cos D}{2\cos C\sin D}$$

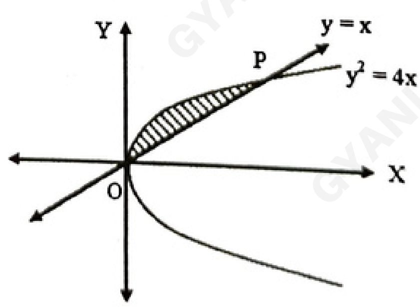
$$\therefore \frac{1}{-\tan A\tan B}=\frac{\tan C}{\tan D} \Rightarrow \tan A\tan B\tan C=-\tan D$$

113.(A)

Refer figure. Point of intersection of given curves are $x^2=4x \Rightarrow x(x-4)=0$

$\therefore O\equiv(0,0)$ and $P\equiv(4,4)$

Required area is shaded.



$$\therefore A=\int_0^4\left(\sqrt{4x}-x\right)dx=2\int_0^4 x^{\frac{1}{2}}dx-\int_0^4 x\,dx$$

$$=2\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^4-\left[\frac{x^2}{2}\right]_0^4=\left(\frac{4}{3}\right)(4\times2)-\frac{16}{2}$$

$$=\frac{32}{3}-\frac{16}{2}=\frac{16}{6}=\frac{8}{3}\text{ sq. units.}$$

114.(B)

$$y(1+\log x)\left(\frac{dx}{dy}\right)-x\log x=0$$

$$\therefore \int\frac{(1+\log x)}{x\log x}dx=\int\frac{dy}{y}$$

$$\therefore \int\frac{dx}{x\log x}+\int\frac{dx}{x}=\int\frac{dy}{y}$$

$\therefore$ log (log x) + log x = log y + log c

$\therefore$ log [x log x] = log (y c) $\Rightarrow$ x log x = yc

115.(C)

Negation of $(\forall x \in R, x^2+1=0)$ is $\exists\, x \in R$, such that $x^2+1\neq 0$

116.(D)

The required plane passes through the point (7, 8, 6) and is parallel to XY plane.

$\therefore$ It is perpendicular to z axis and direction ratios of z axis are 0, 0, 1.

$\therefore$ Required equation of plane is

$0(x-7)+0(y-8)+1(z-6)=0 \Rightarrow z=6$

117.(A)

$$F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\therefore\ |F(\alpha)| = \cos\alpha(\cos\alpha)+\sin\alpha(\sin\alpha)=\cos^2\alpha+\sin^2\alpha=1$

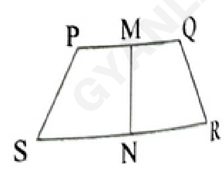
$$\therefore\ adj[F(\alpha)]=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}^T$$

$$=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos(-\alpha) & -\sin(-\alpha) & 0\\ \sin(-\alpha) & \cos(-\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}=F(-\alpha)$$

$$[f(\alpha)]^{-1}=\frac{adj[F(\alpha)]}{|F(\alpha)|}=F(-\alpha)$$

118.(D)

Refer figure.



p, q, r, s, m, n be the position vectors of points P, Q, R, S, M, N.

Now $\overline{PS}+\overline{QR}$

$=\bar{s}-\bar{p}+\bar{r}-\bar{q}\quad=(\bar{s}+\bar{r})-(\bar{p}+\bar{q})$ ...(1)

Since M and N are mid-points of PQ and RS respectively, we write

$\bar{m}=\dfrac{\bar{p}+\bar{q}}{2}$ and $\bar{n}=\dfrac{\bar{r}+\bar{s}}{2}$

$\therefore$ eq. (1) becomes

$\overline{PS}+\overline{QR}=2\bar{n}-2\bar{m}=2(\bar{n}-\bar{m})=2\overline{MN}$

From given data, t = 2

119.(A)

The probability of getting head when a coin is thrown = $\frac{1}{2}$.

The probability of getting a number greater than 4 when a die is thrown = $\frac{2}{6} = \frac{1}{3}$.

Hence required probability = $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{4}{6} = \frac{2}{3}$

120.(A)

From given data, we write

$\bar{a}+\lambda\bar{b} = (2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$ ...(1)

Since (1) is ⊥er to $\bar{c}$, we write

$(2-\lambda)(3)+(2+2\lambda)(1)+(3+\lambda)(2)=0$

$\therefore\ 6-3\lambda+2+2\lambda+6+2\lambda=0 \Rightarrow\ \lambda=-14$

121.(B)

Equation of plane passing through (–2, 2, 2) is $a(x+2)+b(y-2)+c(z-2)=0$

Since this plane also passes through (2, –2, –2), we get

$4a-4b-4c=0 \Rightarrow a-b-c=0$ ...(1)

Normal of the plane is parallel to

$9x-13y-3z=0 \Rightarrow 9a-13b-3c=0$ ...(2)

Solving (1) and (2), we write

$$\frac{a}{\begin{vmatrix}-1 & -1\\-13 & -3\end{vmatrix}} = \frac{-b}{\begin{vmatrix}1 & -1\\9 & -3\end{vmatrix}} = \frac{c}{\begin{vmatrix}1 & -1\\9 & -13\end{vmatrix}}$$

$\therefore\ \frac{a}{-10} = \frac{-b}{6} = \frac{c}{-4} \Rightarrow \frac{a}{5} = \frac{b}{3} = \frac{c}{2}$

Hence equation of required plane is $5(x+2)+3(y-2)+2(z-2)=0$ i.e. $5x+3y+2z=0$

122.(D)

Let $z = a+ib$ and we have

$\frac{b}{a} = \tan\left(\frac{5\pi}{6}\right)$ and $\sqrt{a^2+b^2} = 2$

$\therefore\ \frac{b}{a} = \tan\left(\pi-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = \frac{-1}{\sqrt{3}}$ and $a^2+b^2=4$

$\therefore\ b = \frac{-a}{\sqrt{3}}$ and $a^2+b^2=4$

$\therefore\ a^2+\frac{a^2}{3}=4 \quad \Rightarrow \quad 4a^2=12 \Rightarrow a^2=3 \Rightarrow a=\pm\sqrt{3}$

Also $b = \frac{-a}{\sqrt{3}} = \frac{\pm\sqrt{3}}{\sqrt{3}} = \pm 1$

Since complex number lies in 2^{nd} quadrant,

$a=-\sqrt{3}$ and $b=1 \Rightarrow z=-\sqrt{3}+i$

123.(B)

$$\bar{c} = \bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(9+30) = 39\hat{k}$$

$|\bar{a}| = \sqrt{(3)^2+(-5)^2} = \sqrt{34}$ and $|\bar{b}| = \sqrt{(6)^2+(3)^2} = \sqrt{45}$ and $|\bar{c}| = \sqrt{39^2} = 39$

$\therefore\ a : b : c = \sqrt{34} : \sqrt{45} : 39$

124.(D)

Polar coordinates of $z = a + ib$ are $\left(\sqrt{2}, \frac{\pi}{4}\right)$

$\therefore\ \sqrt{2} = \sqrt{a^2+b^2} \Rightarrow a^2 + b^2 = 2$ and $\tan\left(\frac{\pi}{4}\right) = \frac{b}{a} \Rightarrow \frac{b}{a} = 1 \Rightarrow a = b$

$\therefore\ a^2 + b^2 = 2 \Rightarrow 2a^2 = 2 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

Since point lies in 1st quadrant, $a = 1 \Rightarrow b = 1$

$\therefore$ Cartesian coordinates are (1, 1)

125.(D)

Let (h, k) b the centre of the circle. It lies on the line $x - 4y = 1 \Rightarrow h = 1 + 4k$

$\therefore$ centre $\equiv (4k + 1, k)$

Circle passes through points (3, 7) and (5, 5)

$\therefore\ (4k + 1 - 5)^2 + (k - 5)^2 = (4k + 1 - 3)^2 + (k - 7)^2$

$\therefore\ (4k - 4)^2 + (k - 5)^2 = (4k - 2)^2 + (k - 7)^2$

$\therefore\ 16k^2 + 16 - 32k + k^2 + 25 - 10k = 16k^2 + 4 - 16k + k^2 + 49 - 14k$

$\therefore\ -42k + 41 = -30k + 53 \Rightarrow 12k = -12 \Rightarrow k = -1$

$\therefore$ Centre $\equiv (-3, -1)$

$\therefore$ Radius $= \sqrt{(-3-5)^2+(-1-5)^2} = 10$

Hence equation of required circle is

$(x + 3)^2 + (y + 1)^2 = (10)^2$ i.e. $x^2 + y^2 + 6x + 2y - 90 = 0$

126.(C)

Let $\quad I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 - \sin x \cos x}\,dx$ ...(1)

$$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right) - \cos\left(\frac{\pi}{2}-x\right)}{1-\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 - \cos x \sin x}\,dx \qquad \text{...(2)}$$

Eq. (1) + (2) gives,

$$2I = \int_0^{\frac{\pi}{2}} 0\,dx \Rightarrow I = 0$$

127.(D)
x is element a_{12} in adj (A).

$\therefore$ x = cofactor of $a_{21} = (-1)^{2+1}\begin{vmatrix}0 & 2\\ 2 & 1\end{vmatrix} = -(-4) = 4$

Similarly $y = (-1)^{3+3}\begin{vmatrix}1 & 0\\ -1 & 1\end{vmatrix} = 1 \Rightarrow x + y = 5$

128.(C)
For the shaded region, inequalities are as follows.

$x \geq 0, y \geq 0, 2x + 3y \geq 3, x - 6y \leq 3, 3x + 4y \leq 18, -7x + 14y \leq 14.$

Note :

$-7x + 14y = 14 \Rightarrow 7x - 14 = -14$ and $0 > -14$

129.(D)

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} 1 + \sin\frac{\pi}{2} = 1 + 1 = 2$ ...(1)

$\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} ax + b = a + b$ ...(2)

Since f(x) is continuous at x = 1, we get a + b = 2 ...(3) ...[From (1), (2)]

$\lim\limits_{x\to3^-} f(x) = \lim\limits_{x\to3^-} ax + b = 3a + b$ ...(4)

$\lim\limits_{x\to3^+} f(x) = \lim\limits_{x\to3^+} 6\tan\frac{x\pi}{12} = 6\tan\frac{3\pi}{12} = 6$ ...(5)

Since f(x) is continuous at x = 3, we get 3a + b = 6 ...(6) ...[From (4), (5)]

Solving (3) and (6), we get a = 2, b = 0

130.(A)

Let $I = \int \frac{x^3}{\sqrt{1+x^2}}dx = \int \frac{x^2 \cdot x}{\sqrt{1+x^2}}dx$

Put $\sqrt{1+x^2} = t \Rightarrow 1 + x^2 = t^2 \Rightarrow 2x dx = 2t\,dt$

$\therefore I = \int \frac{(t^2-1)t\,dt}{t} = \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{(1+x^2)^{\frac{3}{2}}}{3} - \sqrt{1+x^2} + c$

Comparing with given data, $a = \frac{1}{3}, b = -1 \Rightarrow a + b = \frac{-2}{3}$

131.(D)

$A^{-1} = \frac{-1}{2}\begin{bmatrix}1 & -4\\ -1 & 2\end{bmatrix} = \begin{bmatrix}\frac{-1}{2} & 2\\ \frac{1}{2} & -1\end{bmatrix}$

$A A^{-1} = I \Rightarrow A\begin{bmatrix}\frac{-1}{2} & 2\\ \frac{1}{2} & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$R_2 \to R_1 + R_2$

$$A\begin{bmatrix} \frac{-1}{2} & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$$A\begin{bmatrix} \frac{-1}{2} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$R_1 \to -2\,R_1$

$$A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

$$\therefore\ 2A + I_2 = \begin{bmatrix} 4 & 8 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix}$$

132.(A)

$$y = \operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) = \frac{\pi}{2}$$

$$\therefore\ \frac{dy}{dx} = 0$$

133.(D)

$$\int_1^4 f(x)\,dx = \int_1^4 [|x-1| + |x-2| + |x-3|]\,dx$$

$$= \int_1^2 (x-1)+(2-x)+(3-x)\,dx + \int_2^3 [(x-1)+(x-2)+(3-x)]\,dx$$

$$+ \int_3^4 [(x-1)+(x-2)+(x-3)]\,dx$$

$$= \int_1^2 (4-x)\,dx + \int_2^3 x\,dx + \int_3^4 (3x-6)\,dx$$

$$= \left[4x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2} - 6x\right]_3^4$$

$$= \left[(8-2) - \left(4 - \frac{1}{2}\right)\right] + \left[\left(\frac{9}{2} - 2\right)\right] + \left[(24-24) - \left(\frac{27}{2} - 18\right)\right]$$

$$= \left(2 + \frac{1}{2}\right) + \left(\frac{5}{2}\right) + \left(\frac{9}{2}\right) = \frac{19}{2}$$

134.(C)

$$\frac{dy}{dx} = \frac{x+2y-1}{x+2y+1}$$

$$\text{Put } x + 2y = t \Rightarrow 1 + 2\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{dt}{dx} - 1\right)}{2}$$

$\therefore \frac{\left(\frac{dt}{dx}-1\right)}{2}=\frac{t-1}{t+1} \Rightarrow \frac{dt}{dx}-1=\frac{2t-2}{t+1}$

$\therefore \frac{dt}{dx}=\frac{2t-2}{t+1}+1=\frac{3t-1}{t+1}$

$\therefore \int\frac{t+1}{3t-1}dt=\int dx$

$\therefore \frac{1}{3}\int\frac{3(t+1)}{3t-1}dt=\int dx \Rightarrow \frac{1}{3}\int\frac{3t-1+4}{3t-1}dt=\int dx$

$\therefore \frac{1}{3}\int dt+\frac{4}{3}\int\frac{dt}{3t-1}=\int dx$

$\therefore \frac{t}{3}+\frac{4}{3}\frac{\log|3t-1|}{3}=x+c_1$

$\therefore \frac{x+2y}{3}+\frac{4}{3}\frac{\log|3(x+2y)-1|}{3}=x+c_1$

$\therefore 3x+6y+4\log|3x+6y-1|=9x+9c_1$

$\therefore 6(-x+y)+4\log|3x+6y-1|=K$

135.(B)

From given data, we write

When $x=0,\ P=\frac{5}{16}=\frac{15}{48}$

$x=1,\ P=\frac{5}{16}=\frac{15}{48}$

$x=2,\ P=\frac{2k}{48}$

$x=3,\ P=\frac{1}{4}=\frac{12}{48}$

Here $\Sigma Pi=1$

$\therefore \frac{15}{48}+\frac{15}{48}+\frac{2k}{48}+\frac{12}{48}=1 \Rightarrow k=3$

$\therefore$ When $x=2,\ P=\frac{6}{48}=\frac{1}{8}$

Now $E=\Sigma p_i x_i$

$=\left[\left(\frac{5}{16}\right)(0)\right]+\left[\left(\frac{5}{16}\right)(1)\right]+\left[\left(\frac{1}{8}\right)(2)\right]+\left[\left(\frac{1}{4}\right)(3)\right]=0+\frac{5}{16}+\frac{1}{4}+\frac{3}{4}=\frac{21}{16}=1.3125$

136.(B)

We have $\frac{dT}{dt}\propto(30-T)$

$\therefore \frac{dT}{dt}=-K(30-T) \Rightarrow \int\frac{dT}{30-T}=\int -Kt$

$\therefore \log|30-T|=-kt+c$ ...(1)

From given data, we write

$\log|30-0| = -10\text{ K} + c$ ...(2)

$\log|30-15| = -20\text{ K} + c$ ...(3)

Solving (2) and (3), we get

$$\log\left(\frac{30}{15}\right) = 10\text{ K} \Rightarrow \text{K} = \frac{1}{10}\log 2$$

Substituting value of K in eq. (2), we get

$$\log 30 = (-10)\left(\frac{\log 2}{10}\right) + c \Rightarrow c = \log 60$$

Thus eq. (1) becomes

$$\log|30-\text{T}| = \frac{-\log 2}{10}t + \log 60$$

$$\therefore \log\left|\frac{30-\text{T}}{60}\right| = \frac{-0.3010}{10}t = -0.03010\,t$$

$$\therefore \frac{30-\text{T}}{60} = e^{-0.03010t} \qquad \therefore \text{T} = -60\,e^{-0.03010\,t} + 30$$

137.(C)

Slope of line $= \tan(90+\propto) = -\cot\propto$

Equation of required line is

$$(y - p\sin\propto) = \frac{-\cos\propto}{\sin\propto}(x - p\cos\propto)$$

$$\therefore (\sin\propto)y - p\sin^2\propto = (-\cos\propto)x + p\cos^2\propto$$

$$\therefore (\cos\propto)x + (\sin\propto)y = p$$

138.(C)

$|\bar{a}+\bar{b}|^2 = |\bar{a}-\bar{b}|^2 + 4\,\bar{a}\cdot\bar{b}$ ...(1)

Now $|\bar{a}-\bar{b}|^2 = |\bar{a}|^2 + |b|^2 - 2\bar{a}\cdot\bar{b}$

$\therefore (5)^2 = (3)^2 + (4)^2 - 2\bar{a}\cdot\bar{b} \Rightarrow \bar{a}\cdot\bar{b} = 0$

Substituting in (1), we get

$|\bar{a}+\bar{b}|^2 = |\bar{a}-\bar{b}|^2 \Rightarrow |\bar{a}+\bar{b}| = 5$

139.(C)

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$$\therefore \tan^{-1}\left[\frac{\left(\frac{x-1}{x-2}\right)+\left(\frac{x+1}{x+2}\right)}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$$

$$\therefore \frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)} = \tan\frac{\pi}{4}$$

$$\therefore \frac{(x^2+x-2)+(x^2-x-2)}{(x^2-4)-(x^2-1)} = 1$$

$$\therefore 2x^2 - 4 = -3 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

140.(B)

$f(x) = 3[x] + 5\{x + 1\}$

$x = -1.32 \Rightarrow [x] = [-1.32] = -2$

Also $x + 1 = -1.32 + 1 = -0.32$

$\therefore [x + 1] = [-0.32] = -1$ and $\{x + 1\} = 0.68$

$\therefore f(x) = 3(-2) + 5(0.68)$

$= -6 + 3.4 = -2.6$

141.(A)

We have $\frac{dr}{dt} = 4$ cm/sec and $A = \pi r^2$

$\therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\therefore \frac{dA}{dt} = 2\pi(10)(4) = 80\pi$

142.(A)

$f(x) = \cot^{-1} x + x$

$\therefore f'(x) = \frac{-1}{1+x^2} + 1 = \frac{-1+1+x^2}{1+x^2} = \frac{x^2}{1+x^2}$

Here $x^2 \ge 0 \Rightarrow \frac{x^2}{1+x^2} \ge 0$

Hence f(x) is always increasing.

143.(C)

$$\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$$

$$= \lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{(x-1)(2x+3)} = \lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)(2x+3)}$$

$$= \lim_{x\to 1} \frac{2x-3}{(\sqrt{x}+1)(2x+3)} = \frac{-1}{2(5)} = \frac{-1}{10}$$

144.(C)

Let $u = (\log x)^x$

$\therefore \log u = x \log[\log(x)]$

$\therefore \frac{1}{u}\frac{du}{dx} = \frac{x}{\log(x)} \times \frac{1}{x} + \log(\log x) = \log(\log x) + \frac{1}{\log x}$

$\therefore \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right]$ …(1)

Let $v = \log x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$ …(2)

$\therefore \frac{du}{dv} = (\log x)^x (x)\left[\frac{1}{\log x} + \log(\log x)\right]$ …(From (1), (2))

145.(A)

Let $I = \int e^{\tan x}\left(\sec^2 x + \sec^3 x \sin x\right) dx = \int e^{\tan x}\left(\sec^2 x\right)(1+\tan x)dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt \quad \therefore \quad I = \int e^t(1+t)dt = e^t(t)+c = e^{\tan x}(\tan x)+c$

146.(C)

$2x^2 - 5xy + 2y^2 = 0$

$\therefore \quad 2x^2 - 4xy - xy + 2y^2 = 0 \Rightarrow 2x(x-2y) - y(x-2y) = 0$

$\therefore \quad (2x-y)(x-2y) = 0$ Thus lines are $2x - y = 0$ and $x - 2y = 0$

Distance of point $(2, -1)$ from these two lines are respectively

$\left|\frac{(2)(2)+(-1)(-1)}{\sqrt{4+1}}\right|$ and $\left|\frac{(1)(2)+(-1)(-2)}{\sqrt{1+4}}\right|$ i.e. $\frac{5}{\sqrt{5}}$ and $\frac{4}{\sqrt{5}}$

Hence required answer is $\frac{5}{\sqrt{5}} \times \frac{4}{\sqrt{5}} = 4$

147.(C)

We have $n_1 = 5, \sigma_1{}^2 = 4, \bar{x}_1 = 2$ and $n_2 = 5, \sigma_2{}^2 = 5, \bar{x}_2 = 4$

Here combined mean $= \bar{x}_c = \frac{(5)(2)+(5)(4)}{(5)+(5)} = 3$

$d_1 = \bar{x}_1 - \bar{x}_c = 2 - 3 = 1$ and $d_2 = \bar{x}_2 - \bar{x}_c = 4 - 3 = 1$

Combined variance $= \frac{n_1\left(\sigma_1^2 + d_1^2\right) + n_2\left(\sigma_2^2 + d_2^2\right)}{n_1 + n_2} = \frac{5(4+1)+5(5+1)}{5+5} = \frac{11}{2}$

148.(C)

Number of necklaces formed from 8 different pearls $= \frac{(8-1)!}{2} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$

149.(D)

We will go by options

(A) $(p \vee q) \vee r \equiv (T \vee T) \vee F \equiv T \vee F \equiv T$

(B) $(p \wedge q) \to r \equiv (T \wedge T) \to F \equiv T \to F \equiv F$

(C) $(p \to r) \to q \equiv (T \to F) \to T \equiv F \to T \equiv T$

(D) $(p \leftrightarrow q) \to r \equiv (T \leftrightarrow T) \to F \equiv T \to F \equiv F$

150.(D)

$\frac{x^2}{a^2} + \frac{y^2}{4} = 1$

$\therefore \quad \frac{1}{a^2}2x + \frac{1}{4}2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \left(\frac{-x}{a^2}\right)\left(\frac{4}{y}\right)$ ...(1)

Also $y^3 = 16x$

$\therefore \quad 3y^2\frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{16}{3y^2}$ ...(2)

Since curves intersect orthogonally, from (1) and (2), we write

$\left(\frac{-x}{a^2}\right)\left(\frac{4}{y}\right)\left(\frac{16}{3y^2}\right) = -1$

$\therefore \quad \frac{64x}{3a^2y^3} = 1$ and we have $y^3 = 16x$

$\therefore \quad \frac{64x}{3a^2(16x)} = 1 \Rightarrow a^2 = \frac{4}{3}$